On the Nature of Spacetime:

General Theory of Relativity vis-à-vis Quantum Theory

To decipher fundamental postulates and approximations inherent in General Theory of Relativity (GTR) and hence in Special Theory of Relativity (STR) as well in Newtonian Theory (NT), we scrutinize and interpret fundamental concepts like events, world-lines and metric tensor of space-time model of GTR. Ubiquitous concept of Action is interpreted in light of this scrutiny and Hilbert Action is derived with this interpretation. It is shown that concept of Entropy in Thermodynamics is fundamentally at variance with these basic postulates of GTR. We show how certain features of Quantum Systems arise by dropping these postulates and approximations of GTR. Though, whether all *the features of Quantum arena can be explained by these, remains an open question.*

1. Introduction:

Ironically today's physics has *two* theories to represent its understanding of natural phenomena. One is Classical Physics – represented in full generality by General Theory of Relativity (GTR) while another is Quantum Physics – represented by Quantum Mechanics (QM) and Quantum Field Theory (QFT). Seemingly irreconcilable fundamental differences in these are well known and documented in numerous studies and may not be repeated here. To appreciate certain basic postulates and approximations of GTR that are not respected by quantum realm and thus may be at heart of these differences is the main aim of current study.

As is well known, another trouble with physics is that both of its theories deploy fundamental equations that are without *rigorous* mathematical derivations. Whether it is Einstein's Field Equations for GTR or Schrodinger's wave equation or Dirac equation for Quantum realm, these are basically heuristically proposed and are taken to be correct as these work wonderfully well in their respective domains. If we can say so, third trouble with physics is that concept of Action, common to both theories, is without physical interpretation yet and is embraced for its utility for leading to equations of physics through a standard prescription. In this study we will be led to an interpretation of physical meaning of concept of Action that is common to both the theories. With this interpretation we will be able to fill few steps in the derivations of basic equations of these theories.

Historically Newtonian Theory (NT) was expanded by Special Theory of Relativity (STR) and later was subsumed by GTR. And this is the sequence in which these theories are introduced in any orthodox treatment. In the current study we begin in the reverse order, however. We begin with final model of space-time in GTR and try to scrutinize fundamental concepts of it to discern core postulates and approximations that are naturally inherent in STR and NT also. We will then move on to drop these postulates and approximations one by one and decipher what features of quantum realm may be explained thus.

We will begin in next section with scrutinizing basic concepts of *events* and *world-lines* in GTR leading to appreciation of space-time of GTR as a *sprinkle* of events on a four-dimensional manifold. We appreciate physical meaning of Metric Tensor as well as gain understanding of scale of spacetime at which GTR works, telling us difference between *absolute* vacuum and what we christen as *Einsteinian* vacuum. Physical meaning of certain aspects of metric tensor that are assumed a priori in GTR becomes evident. Section 3 will present our understanding of *physical* meaning of Action that translate into Action functionals of GTR, STR and NT along any path of evolution. Same understanding of physical meaning of Action leads to derivation of Hilbert's Action functional for gravitation when applied to regions of spacetime. Relation between Action and metrical properties of chosen coordinate system through energy-momentum tensor becomes evident and leads to derivation of celebrated Einstein Field Equations (EFE) of GTR. In Section 4, we summarize postulates and approximations of GTR discerned from scrutiny of earlier sections. We also realize certain facets like reversibility, deterministic nature of evolution and distinguishability of elements that systems modeled on GTR display due to these postulates and approximations. In Section 5, we study Thermodynamics by appreciating that it does not model its systems respecting postulates of GTR: entropy, as a concept of Thermodynamics is shown to come to existence due to this non-conformity. Section 6 gives examples of certain other systems that ab-initio reject fundamental postulates of classical physics and thus demand new models of physics. This will make us appreciate origins of some differences of Quantum physics and Classical Physics. In Section 7 and 8 we drop approximation of continuity and postulate of synchronizability, respectively and give heuristically how features of such systems are parallel to what we find in quantum systems. A basic mathematical treatment is given in Section 9 to such systems and Quantum Diffusion Coefficient of E. Nelson and Quantum Potential of D. Bohm are shown to arise out of this. We summarize briefly in Section 10 and hint at directions for further studies.

2. Events, Worldlines and Metric Tensor:

After STR had mixed up space and time periods between *events* into invariant spacetime period, Minkowski propounded use of four-dimensional spacetime arena for physics to model its rules. He named *each* location of the mathematical manifold that represents spacetime as event and sequence of locations that an entity occupies as *worldline* of the entity. These two terminologies were continued while generalizing the spacetime manifold of Minkowski in GTR.

Einstein's endeavor in GTR was to do away with the restriction of applicability of STR and NT to only inertial reference frames, respecting Lorentz transformation rules and to get to general laws of nature that will hold for *all imaginable systems of coordinates* [1] that may be chosen by any given observer or observers and thus respect most general transformation rules. This complete freedom to choose a coordinate system by any given observer meant that coordinate differentials will, in themselves, not be able to represent any physically significant periods of space or time. This necessitated introduction of an additional mathematical structure - Metric Tensor - that is to be used in conjunction with coordinate differentials to give physically meaningful space and time periods that combine to spacetime periods. Using this construct Einstein could generalize Minkowski spacetime of STR and get a playground for GTR. We begin with scrutinizing these basic conceptions and terminologies used by GTR and thus whole of classical physics[1].

Commencing with conception of *events*, we realize that not much discussion generally follows on use of this terminology in most of the discourses. Deeper scrutiny will tell us many of the assumptions that creep knowingly and unknowingly in the spacetime model of classical physics through this seemingly innocuous term.

Consider first *observability* facet of events. At the very outset, we realize that in Einstein's conception of spacetime manifold not all locations are expected to be observable for an observer. This makes, by extension of fact that all locations are being termed events, some of the events are unobservable in standard conceptions. To appreciate this, consider Einstein's statement –

All our space-time verifications invariably amount to a determination of space-time coincidences. If, for example, events consisted merely in the motion of material points, then ultimately nothing would be observable but the meetings of two or more of these points. [1]

This paragraph introduces another terminology – *spacetime coincidences*. All material entities exist at a certain spatial location and at a particular time vis-à-vis a given observer, as numbered by quadruplet in a coordinate system of choice of the observer. When any observation is made of any material entity, we need another material entity that is being used for the purposes of measurement to be thereat, at that space location at that time, given by same quadruplet. That is, nothing is observable except when there is a spacetime coincidence of two or more material entities. Though this talks about deliberate measurement by an observer, it is understood that any happening that is conveyed to another entity has to be a spacetime coincidence of more than one entity. We call all these conveying as observations – conscious or not, deliberate or not.

We proceed severely with inclusion in our abstraction of physics through mathematics only what is observable– spacetime coincidences of Einstein.[2] Nothing else is observable then collection of these coincidences and thus nothing else should be material for doing physics. By extension, events along single worldlines that lie between two consecutive spacetime coincidences should have no role to play in any theory of physics. These may have mathematical individuations through some arbitrarily drawn mathematical net of coordinates but these will have no meaning for physics of the spacetime. So, events along the worldline between two spacetime coincidences, if we take Minkowski's terminology of events for *all* locations of spacetime are of no physical meaning!

We however appreciate that *an event not being an input to physical laws* is quite a paradox and thus use of the terminology *event* for locations not being spacetime coincidence is wrong, misleading and a misnomer.

Beyond the issue of observability and thus being an input to physical laws, basic *nature* of an event also indicates that it must be a spacetime coincidence. An event intuitively leads to change in something and vice versa - any change is necessarily an event too.

[1] We do not take here Thermodynamics as classical physics despite its date of origin. Reasons for this will become clear in the monograph.

[2] Here we are deviating from Einstein who gave only epistemological importance to observables and not full ontological importance. He stated that though any theory must produce observable results of all experiences but must also provide a logical explanation of underlying reality which may not be directly observable in itself.

What is the change in *mere* motion of material entities between spacetime coincidences with other material entities? It is but off course only *location* on the spacetime manifold. With approximation of worldlines of particles and light being infinitely densely packed in manifold representation of spacetime, Einstein individuates every location on the manifold with events as falling on some worldline. And this mathematical individuation then lends meaning to *change in location* and through this change to *event*. This is circular- change of spatiotemporal location of an entity counts as a change that is to be termed event, and every location is considered individuated and differentiated by christening it as an event. And how every location between spacetime coincidences is being individuated mathematically? It is by assuming a *straight (geodesic)* path between the two coincidences i.e. by assuming a *shape* of the path that is unobservable.

We would like *change* and thus the word *event* to mean differently and in concrete physical terms. For us, an event has to be appreciated as such by the entity experiencing the event. This means an event should necessarily be a spacetime coincidence involving two or more physical entities. That is, for us *an event is necessarily a spacetime coincidence*, an Einsteinian observable. In contrast *locations as well as worldline between two consecutive coincidences* are no-change scenarios for attributes of an entity and are unappreciable as any event by entity itself and hence devoid of physical meaning of an event.

We equate then events with spacetime coincidences and distinguish locations and events.

We thus have locations on spacetime without events thereat. Worldlines are extrapolations between spacetime coincidences: like an imaginary thread connecting beads. We do not know, and *cannot* know, the *shape* of worldline between two consecutive coincidences and whatever mathematics we do on this manifold, it must allow any shape and hence trajectory between two coincidences without impacting physics.

At this juncture, it may be felt that, if we are equating events with spacetime coincidences exactly, why do we need to change the standard nomenclature of each location of spacetime manifold as events and why cannot we continue with it and deploy nomenclature of 'spacetime coincidences' to mean physical events in our discourse? We understand this might look desirable to keep the standard notation, but word event is so physical in itself and conveys the fact that something physical has happened when we speak of an event, that it is desirable to use physically intuitive nomenclature of event as a physical happening and restricting its use to an observable spacetime coincidence.

How do we model collection of events in a spacetime manifold once we refuse to equate all locations as events? To mark spacetime coincidences or events we must continue to use coordinates – that is collection of quadruplets of numbers that are assigned uniquely to each event in such a way that nearby events have nearby values. *In christening events with coordinates, one demands* smoothness *but foregoes every thought of mensuration* [2]. Thus what we have is – lot of *flags* at *discrete* locations, each having a set of four numbers written on it; *voids* in between have nothing – *nothing that may distinguish or individuate a part of the void with another*. We draw up *coordinate* lines in the voids as *extrapolated* mathematical net but this mathematical identification of locations in voids has no physical significance.

We may, then, while continuing to model spacetime as continuous four-dimensional manifold, model events (spacetime coincidences) as *sprinkled* on this manifold.

Just to make this difference between our and Einsteinian conceptions vivid, we reproduce in figure 1 a picture of maze of worldlines from [Gravitation - MWT] and in figure 2, same pictures with only flags. For us second picture is what is available to an observer to deduce physics. First picture misrepresents reality in the sense that it extrapolates and *draws* worldlines between events that are physically unobservable.[3]

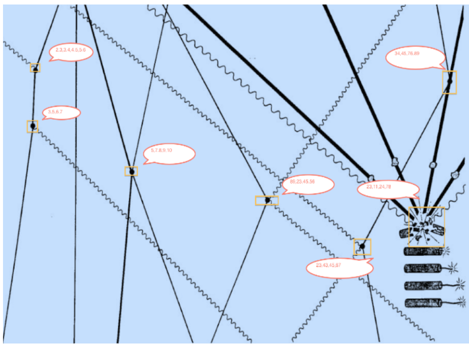

Figure 1

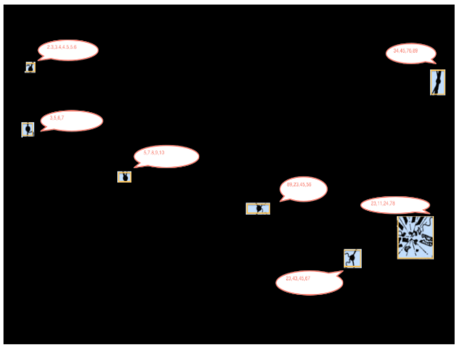

Figure 2

Thus, in effect we are going with following picture of sprinkle of events on spacetime manifold (Stanford):

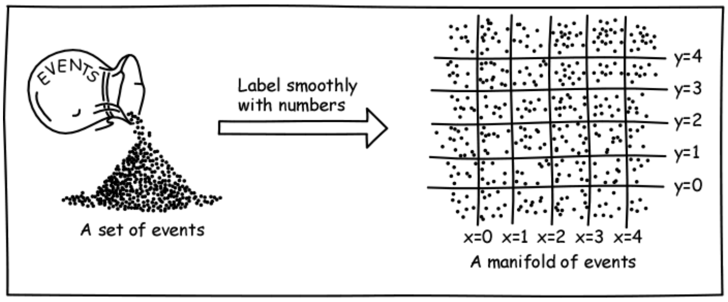

Figure 3.

Though coordinate systems are very convenient tools for thinking and modeling physics' rules, our demand has only been unambiguous nature of allocation of quadruples to events and smoothness. Even mensuration is not allowed on the basis of coordinate system alone, as it would assume meaningful distances over mathematical net between physically individuated locations. As we have indicated before, we must add a *structure* (Metric Tensor) on these coordinate systems that indicate physically meaningful aspects of distribution of these flags. Now we turn to this structure.

As we understand, there are two ways different coordinates systems get chosen. One, a single observer may choose to change coordinate system it is using leading to change of only mathematical representation of exactly the same physical reality. That is exactly same sprinkle of the events is getting represented by different set of quadruplets by the same observer. Second, we may have different observers, each using one's own chosen coordinate system and we need to translate between the two.

[3] If a worldline is seen in cloud chamber then it is not a mathematical extrapolation between events but actually a series of events – a particle whose world line is being observed in cloud chamber is actually participating in those many events in succession to display worldline. Here we are talking about worldline that is unobservable due to existence of no physical events along it to mark it.

In former case of change of coordinate system, mathematical structure that represents some physical aspect of the reality within any physically delineated region of manifold must give exactly the same invariant output. We may call this *Mathematical Equivalence* – as it is equivalence between two mathematical models of same physical reality as perceived by same observer. It is trivial to appreciate this fact.

In later case of change of coordinate system while moving from an observer to another, we should be very careful about what we demand invariance of – it is the same physical reality being perceived by two different observers differently – they are going to have different space and time periods between observed events and thus sprinkle will get *skewed*. Invariances of certain aspects of distribution of events in such a scenario should be christened *Physical Equivalence* – as it is equivalence of physical aspects of observations of two different observers.

A fundamental aspect of sprinkling of events on spacetime manifold must be that of *number* of events that are counted in a given *region* or along a given *path*. In fact, giving these numbers for all arbitrary paths and all arbitrarily chosen volumes, along with physical parameters of the entities involved in these events must end up specifying all that is physical about the spacetime manifold and thus *whole endeavor of physics should be to appreciate these*.

We first look at the pure number aspect of the sprinklings. We realize that this number, both for an arbitrary region as well as any given path, is invariant in GTR for all observers: a physical equivalence. Einstein indicates this: *These points of intersection naturally are preserved during all [coordinate] transformations (and no new ones occur) if only certain uniqueness conditions are observed. It is therefore most natural to demand of the laws that they determine no more than the totality of space-time coincidences.*[4] [1]

'Points of intersection' are same as spacetime coincidences for Einstein and in our terminology these are same as events.[5] This fact of invariance of numbers of these along various paths and in different volumes is trivial in case of change of coordinate systems by the same observer. In case of transformation between coordinate systems deployed by two observers wherein we are dealing with same reality being perceived by observers who are differently placed vis-à-vis each other, will they always find equal number of events along a curve or in a given volume of the manifold? A curve will change its shape for an observer but what about number of events along it? A given volume basically means that bounded by a given *hypersurface.* The *shape* of this hypersurface boundary of the region will surely change, but what about number of spacetime coincidences inside it? Will these numbers remain the same for different observers, is the question. Einstein, via above quote answers this in affirmative.

All classical theories before General Theory of Relativity had answer as yes. In case of Newtonian spacetime, it is trivially so. Spacetime arena is *passive* and *unchanging* among observers. Nothing changes in the manifold (which is further split into unchanging space and time arenas)- not even shape of the hypersurface boundary of

[4] Here we note that Einstein is speaking of physical laws as limiting to observables. Though detail reading of his writings and quotes give away his intuitive belief in under laying reality behind creation of observable events, especially in 1915 GTR paper he emphasizes physical importance of only observables. This dichotomy is, as I understand, due to his belief in his final equations and general covariance but failure to find a satisfactory solution of *hole* argument for himself.

[5] In fact expressions 'sense experiences', 'points of intersection' and 'spacetime coincidences' have been variously used by Einstein to mean the same – what we christen events in this monograph.

any region. What about Special Theory of Relativity? Two events may have different time and space periods between them for two observers but they *remain two.* In fact spacetime distance between them remain constant even if periods (space or time) do change themselves. One of the periods may become even zero, but the spacetime distance is preserved, meaning thereby that these remain two distinct events in the spacetime manifold. This invariance vividly tells us that, in STR too, number of events in any given region of spacetime shall remain same for any two observers who find this region within their light cone.

Einstein, as the quote above tells us, believed so to be true in case of GTR. Physically this is intuitive – while moving from an observer to another, events observed are neither dropped nor added. GTR goes ahead with the *postulation* that along any given path or in any given region of spacetime manifold, total number of events, or Einsteinian spacetime coincidences, shall remain equal for all the observers.

Let's count these events along any path. In GTR we have metric tensor that gives *invariant path length* among any arbitrarily chosen coordinate systems by any observer as well as among coordinate systems chosen by different observers. This length off course is given in units of space or time and not in *pure* number. But its invariance among all coordinate systems that may be deployed to represent spacetime hints at an interpretation of metric tensor in GTR:

Metric tensor encodes *number of events (spacetime coincidences) on the spacetime manifold. It* indicates*, through invariants it forms in conjunction with coordinate periods, spacetime coincidences that are encountered when we traverse any curve.*

Let's take the length of a curve in terms of metric tensor: $ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$. In this expression ds represents spacetime period, dx^α and dx^βrepresent coordinate differentials while $g_{\alpha\beta}$ represents metric tensor components and Einstein's summation convention is in operation[6]. As stated, dimension of this quantity ds is different than pure number and thus this quantity cannot *directly* represent this number; but it shall be the *representative* of this number. Hence, the use of words like encodes or indicates. Clearly this expression will have to be multiplied by appropriate factor of dimension of inverse of space (or time) to get to a pure number. It should also be clear that this factor itself will have to be invariant among observers as final number is to be invariant and ds is invariant in itself. What is this factor that gives spacetime coincidences or events directly in conjunction with ds will be looked into when we come to conception of Action. For now, we say that metric tensor *encodes* events' numbers along a path and this expression ds *indicates* these numbers.

There is another very strong hint to this interpretation. Spacetime distance along any path in spacetime formed with metric functions and coordinate periods gives *proper time* experienced by an entity that is undergoing the motion along the path. This flow of proper time must be related to number of events that this entity encounters along its evolutionary path as fundamentally each event has to be a tick of the clock and vice versa. Without an event there cannot be an appreciation of flow of time by the entity and an event cannot go without appreciation of flow of time. Explicitly, there is no appreciation of time by an entity between spacetime coincidences or events – proper

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[6] If a parameter, like α, is repeated, once as upper script and once as lower script, we take sum of all terms with all values of this parameter, α.

time counter increases by *one unit* for each spacetime coincidence encountered. That is, spacetime distance measured along any curve being *proper time measured by the entity going along the curve* is nothing but an indication of number of events – spacetime coincidences - that are encountered along that curve by this entity. Again, proper time being in dimensions of time, proportionality factors shall be required of proper dimensions to convert path length ds into exact measurement of proper time. Proportionate and directly related however these two would be.

What about number of events in a given *volume* of spacetime? Along a path, expression for ds captures geometrically all that is required (along with an invariant factor) through integration along the path. In case of volume, we of course realize that coordinate volume differential $dx^0 dx^1 dx^2 dx^3$cannot indicate this physically meaningful number. We have another expression for *geometric* volume of spacetime: $d\tau = \sqrt{g}\, dx^0 dx^1 dx^2 dx^3$. Here 'g' is (positive modulus of) determinant of metric tensor. Though this expression is invariant among all allowed coordinate systems, it will be seen later that this expression will have to be multiplied with another geometric invariant – capturing the *Riemannian Curvature* of the infinitesimal region – along with another invariant factor of requisite dimensions before integrating over spacetime region to throw the invariant number of events in that region.

This appreciation of meaning of metric tensor immediately throws at us two important conceptual issues that must be addressed forthwith.

First: This issue is exactly like that in kinetic theory of gases. Though we should have *number* of molecules in any volume as integer valued functions, in approximation, we take these numbers to be represented by *real* valued smoothly varying functions that when multiplied by coordinate infinitesimal volumes give these numbers. This works due to humongous numbers of molecules involved in any given volume however minute. Similarly, in approximation, we can consider our sprinkling of the events to be encoded by real and smoothly varying functions- $g_{\alpha\beta}$. Only when we reduce the scales to Planck scale regime or thereabout our modelling of number of events along any curve by real and smoothly varying functions will fail and we will have to have discrete functions. Same as we expect ideal gas kinetic theory formulation to fail in gas becomes too rarified.

Second: We know that in GTR, we are not allowed to model metric functions to have zero values in any finite volume of spacetime. A non-zero value indicates non-zero events. Events are spacetime coincidences, happenings to material entities. Thus, non-zero events in any finite region means non-zero matter in any finite volume, however small, at all locations in the spacetime manifold. Ideal vacuum, that is *complete* absence of matter, then, is a no-no in GTR! Lets note that in Kinetic Theory of Gases also we do not model *exact* vacuum – there is no finite location inside the container where there are zero molecules exactly– if we did this, zero molecules in some finite volume at any location inside the container would have created zero pressure situation there and model would have led to a singularity. This be so, we still talk about *mean free path* for molecules to travel along! These approximations, however crude they might feel when looked at this way, work perfectly for classical physics due to humungous number of individual entities involved in any scale at which we do these calculations. To impart more confidence in this kind of modeling in GTR, one may note that Planck's scale is so much more smaller than molecular sizes and numbers of events involved here are very large compared to number of molecules that kinetic theory applies itself to.

There is something more to this approximation in GTR however. In standard accounts of GTR, we have Minkowski metric, generally represented by $\eta_{\alpha\beta}:=(-1,1,1,1)$ (a diagonal 4 by 4 metric with '0' as subscript for time coordinate and 1,2 and 3 for space coordinates) as representing the *vacuum*. That is, when we move away from all *ponderable* mass (generally asymptotically) we approach this metric. Also, we never have Minkowski metric valid in any *finite* region exactly. So what represents absolute vacuum $g_{\alpha\beta}:=(0)$ or $g_{\alpha\beta}=\eta_{\alpha\beta}$?

As we have realized $g_{\alpha\beta}:=(0)$ means no events whatsoever and thus no matter whatsoever. This complete or absolute vacuum is a no-no in GTR. Coming to Minkowski metric $\eta_{\alpha\beta}:=(-1,1,1,1)$, it is easy to see that it actually represents a situation of uniform sprinkling of events on the spacetime manifold. This however is *not* absolute zero matter – just uniformly distributed matter. It is clearly unphysical to expect this to be true over a finite region. We may model actual sprinkling as a variation over this *base* pattern of sprinkling - $g_{\alpha\beta}=\eta_{\alpha\beta}+h_{\alpha\beta}$. Then neither $g_{\alpha\beta}:=(0)$ nor $h_{\alpha\beta}=(0)$ over any finite region.

What we generally do in models of GTR is that we take only huge *ponderable masses* into account and neglect matter in vast expanses of universe in between for purposes of affecting the gravitational field. But this expanse does not have *zero* matter. In fact, these matters do undergo events given by metric tensor components thereat. In these expanses of negligible or let's say imponderable masses, we have metric tensor $g_{\alpha\beta}$ varying very little from Minkowski metric $\eta_{\alpha\beta}:=(-1,1,1,1)$ but not exactly same as this metric. This metric then represents a uniform sprinkling of gravitationally negligible mass and thus in some models of universe, we approach this Minkowski metric asymptotically, away from ponderable or masses of significant value for gravitational influences. We can christen this region as *Einsteinian vacuum*- a region wherein we don't have *ponderable* mass but not ideal or complete vacuum.

Difference between STR and GTR comes from STR assuming Einsteinian vacuum throughout the universe with uniformity of sprinkling of events throughout. In GTR we have variation in sprinkling in this Einsteinian vacuum too - we do have field equations giving metric tensor in these regions as well as *gravity waves* being modeled to be propagating through these.

When we move to STR, we approximate the sprinkling by $\eta_{\alpha\beta}$. This strips STR from ability to represent variation of sprinkling directly. This is fundamentally the only difference in GTR and STR – GTR models fully non-uniform sprinkling while STR restricts itself to uniform sprinkling. This tells us origin of gravitation too – it originates in non-uniformity of sprinkling of events on spacetime manifold. When STR needs to tackle gravitation it has to add another layer of a field representing gravitation over its base manifold.

One vivid way to visualize this picture of spacetime with lumps of matter at certain locations and negligible at approximated vacuum, is to look out of the window of an airplane in the night to observe a city. Only lights are visible with black voids in between. These light locations are akin to appreciable mass on a manifold with voids representing effective vacuum. We know from experience that there is physical matter between these lights, but these are non-ponderable due to not enough light being coming out of these areas (very small density leading to not-enough events thereat). To emphasize this more, see following satellite picture of India in night portrays the sprinkling model of appreciable mass on a manifold.[7]

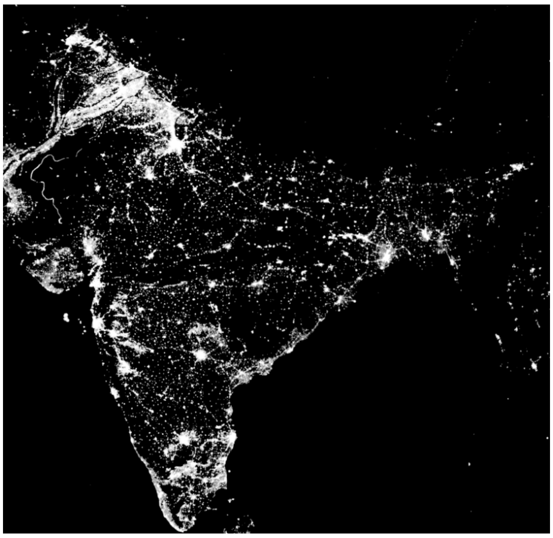

Figure 4:

Let's appreciate the levels of models. Figure 2 is the deepest model stating that at Planck's scale or thereabout we have individual events sprinkled about on a manifold. Gaps between these events are *actually devoid of events* (in our sense i.e. devoid of spacetime coincidences)[8]. We need to physically model this situation of *discrete* when we take our scales of experiments to such a small realm. Classical physics, including GTR, does not do physics at this scale. It does at a much higher scale where we have assumption of events at each location becoming plausible. We however neglect matter, even at these higher scales, in vast expanse that we model as *effective* or *approximate or pseudo* vacuum - Einsteinian vacuum- of classical physics as an added approximation. We however do not allow *uniformity* of distribution of even this matter in Einsteinian vacuum in GTR as mathematically variation in this metric is going to be indicative of gravity. We have non-zero and varying (generally modelled as small perturbation over Minkowski metric) metric in this approximate vacuum in GTR even when there is no matter modelled herein through matter tensor.

Wherever we have clusters of appreciable mass, there are going to be much more events and thus we will have higher values of metric functions. As we move away from these locations, into Einsteinian vacuum locations, we have less and less events and here we model in GTR small perturbations about Minkowski metric tensor. As these metric functions are used to calculate distances in the spacetime, this vividly tells us *how matter tensor is telling geometry how to curve*. To appreciate the other side of the coin of GTR – *how geometry dictates to matter how to move* – we now proceed to give an interpretation of *Action* function.[9] From this interpretation we will recover celebrated Einstein's Field Equations.

[7] On occasion of Diwali – a festival of light!

[8] What is meant by actual vacuum of this kind is still a metaphysical question – quite outside the ambit of this discourse.

[9] As a function of path it's actually termed functional.

Before we move on however we must state one impact of having non-zero metric tensor throughout. This non-zero value mean that we will have non zero events everywhere and thus we get back to standard approximation of calling each and every location as events! Reader may feel that we need not have spent so much time on differentiating between locations and events but importance of this differentiation cannot be over emphasized when we move to appreciate basic postulates and approximations inherent in GTR and when we lower the scales of our models or move towards quantum realm.

3. Action and Einstein's Field Equations:

We have interpretation of metric tensor as *encoding* or *representing* number of events through differential of length – ds – along a path in spacetime. We have also noted that to represent actual *dimensionless* numbers of events this path length differential has to be combined with factors of appropriate dimensions. This construct then will give us representation of *number* of events that are traversed along a path. We have also commented that an invariant construct out of metric tensor will be required for encoding number of events in a region of spacetime and again an invariant multiplication factor will take this to actual number present therein.

We also realize that for wholesome representation of sprinkling of events, other than numbers of events along any path or within a region, we need also to capture physical characteristics of physical entities that are participating in these events. Combining these two requirements, we appreciate that factors that combine with geometric expressions out of metric tensor to give us pure numbers of events shall be borrowing their constituents from physical parameters of the entities involved in these events. These will teach us how intimate relationships between space and time measurements and physical parameters of entities undergoing these movements are.[10] Kinematics can never be *pure* kinematics.

Searching for these invariant factors takes us to appreciation of physical meaning of Action that is a ubiquitous concept present in both the theories of physics. Despite being around for more than three centuries right since inception of classical theories and despite being at the core of modern quantum theories too, this concept has eluded explanation of its physical meaning. It is accepted because it works! In fact, action functional[11] is generally *guessed* for a particular theory and is considered as the right one if its extremization (through Principle of Minimum Action: PMA) leads to equations of evolution that hold true. PMA itself goes without any fundamental proof, but is postulated.

To begin our treatment of Action, we first recapitulate the way this concept is used to *derive* what we call as Einstein Field Equations (EFE) of GTR. As sprinkling of events over spacetime manifold in numbers and physical parameters of entities involved in these events are the only physically important aspects of our model, we must speak about what do Einstein's equations represent in terms of this sprinkling. In case of Newtonian Physics or STR, we represent sprinkling of events as a *variation* over

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[10] It was never realized before Einstein that space and time periods may be intimately related with physical parameters like mass, charge etc. Einstein's most famous equation $E=mc^2$ was a monumental achievement that came out of blue from Special Theory of Relativity. Kinematics was, before that day, always geometric and distinct from dynamics, a physical discipline.

[11] It is called a functional and not a function as it depends upon the paths of probable evolution. Paths may be considered as given function and thus Action value is dependent on the functions and not on direct parameters as we are generally used to. Also, its extremization requires us to consider change in its value when we vary functions that represent the paths and not, as in regular calculus where we consider variation of parameters.

fundamentally uniform *base* sprinkling captured directly by coordinate grid. In these theories than fundamental metric is uniform – either Euclidean (NT) or Minkowski (STR) and does not require any field equations for determination.

Variation or non-uniformity over base sprinkling is captured in these theories in various *potential* functions that are additional structures over this base.[12] Fact that non-uniformity of metric tensor is endowing GTR with capability of representing gravitational field without requiring any further field over spacetime, informs us that metric tensor components themselves represent gravitational potentials in GTR (to be exact, non-uniform components of metric tensor i.e. $h_{\alpha\beta}$ of the decomposition $g_{\alpha\beta}=\eta_{\alpha\beta}+h_{\alpha\beta}$ represent these potentials).

Let's first write down Einstein Field Equations (EFE):

$$G^{\alpha\beta}=R^{\alpha\beta}-\frac{1}{2}g^{\alpha\beta}R=\kappa T^{\alpha\beta}. \quad (1)$$

Here G represents Einstein tensor, $R^{\alpha\beta}$ represents Ricci tensor, scalar R represents Riemannian curvature scalar and $T^{\alpha\beta}$ represents energy-momentum tensor for the material present in spacetime. κ represents a constant that generally is taken in such a way that in case of weak gravitation approximation, we get EFE to reduce to Newtonian gravitational law.

Though there is no rigorous proof[13] of this equation generally there are two approaches – first one, pioneered by Einstein[14] rests on physical intuitions along with certain mathematical requirements derived out of principle of relativity or equality of all observers while second one takes a 'Royal route' of application of Principle of Minimum Action (PMA) on the action functional first proposed by Hilbert. We wish to define Action in a way that leads to EFE and thus we take second route here.

This route is sometimes called a royal way of deriving these equations as it involves lesser tortuous ways and heuristic reasoning. But it begins with a (royal) guess - Hilbert Action functional:

$$S=\int R d\tau=\int R\sqrt{-g}\,dx^0\,dx^1\,dx^2\,dx^3. \quad (2)$$

As indicated no physical reason is to be given for choosing this function.[15] Boundaries of integration are completely arbitrary in the integral expression. Following standard method of variation, we vary this function S with respect to changes in metric functions to reach

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[12] When we move from EFE to Newtonian limit (to check weather these reduce to Newton's hugely successful gravitation law in approximation of weak and static (not changing with time) gravity and movements of entities at speeds considerably lower than that of light) we get variation of metric $g_{\mu\nu}$ from Minkowskian metric $\eta_{\mu\nu}$ i.e. $h_{\mu\nu}=g_{\mu\nu}-\eta_{\mu\nu}$ as gravitational potential: $h_{00}=-2\Phi$. Thus these potentials capture the non-uniformity of sprinkling over Minkowskian grid.

[13] This must be a shock to all non-physicists that fundamental equation of GTR lacks a formal derivation. This shock turns to awe that fundamental equations of Quantum Physics – Schrodinger Equation and Dirac Equation – share this lack of formal proof.

[14] Einstein had much more tortuous route to final equations and it is too simplistic to say this is the approach that he followed directly. His reasoning has been distilled over the years to this form of argument, but as a pioneer he no doubt had to climb much harder slopes and draw upon undisputedly highest levels of human intellectual capacity.

[15] Though heuristic reasons are do thrown up. Like, it has to be invariant scalar, as simple as possible, should generally be comprising of first order derivatives but as these may be reduced to zero by coordinate transformations it should do next best – be comprised of second order derivatives etc. Though the simplest guess is without R in the integrand and just the determinant term, it leads to natural condition on metric tensor and fails to be a field equation. Guesses without reasoning on physical ground.

$$\delta S = \int \left(R^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} R \right) \delta g_{\alpha\beta} d\tau.$$

It is assumed that *coordinate* boundary of integration is not being varied, only the gravitational field given by metric tensor components. We have also not put any restrictions of the variation in these fields except mathematical smoothness and smallness, including no restriction about changes $\delta g_{\alpha\beta}$ having to be zero necessarily on the boundary. Now we take δS as a linear functional of the infinitesimal $\delta g_{\alpha\beta}$ and hence of the form

$$\delta S = \int T^{\alpha\beta} \delta g_{\alpha\beta} d\tau. \qquad (3)$$

Coefficient $T^{\alpha\beta}$ is *defined* to be the energy-momentum tensor.

With this definition of energy-momentum tensor we immediately get Einstein's Field Equations (EFE) as given in (1).

Our line of attack would be to first interpret in our model what is called 'Action' in physics and then to proceed with physical postulate of 'Principle of Minimum Action-PMA'. This principle shall be assumed to be true for now and will be commented upon later. For a start, we take Action functional taken along an evolution path by an entity moving in a given gravitational field. This will allow us to realize what Action functional may physically mean. We will then deal with Action function in a given *region* of spacetime with this realization.

As the chosen evolution path taken by the entity, among various probable ones, has to be same for all the observers observing the system, we realize first that Action functional has to be an *invariant* function of paths with respect to all the observers.[16] We also have *number* of events along any path as fundamental invariant of our model. It is natural to surmise that Action might represent this number directly itself. Again, whatever definition we conjure up for Action, we want it to be *applicable to whole of physics, classical as well as quantum*. In this later realm, we have Action as quantized value- some integer multiplied by Planck's constant. This constant carries the dimensions of Action. Now, though numbers of spacetime coincidences or events along various paths are being approximated using real numbers in the scale of spacetime that we do classical physics at, these numbers *are* integers – however humungous.

Given this, we propose the following *definition* for Action and see whether it leads to standard formulations of this concept in physics:

Action function along an evolution path for an entity represents the number of events (spacetime coincidences) encountered along the path multiplied by universal (modified) Planck's constant ($\hbar = h/2\pi$).

We have been doing physics since its birth in terms of space and time periods traversed by an entity along its evolution path. We have had no use for Planck's constant in classical physics. In fact, in the realm of GTR and thus whole of classical physics, we have number of events going to infinity while Planck's constant going to zero. Multiple of the two – number of events and Planck's constant, for an infinitesimal period in

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[16] Its called a functional and not a function, as its value depends upon wholesome path and not on any finite parameters.

spacetime goes together to a real number. And real valued Action functional represents it in classical physics.

This real number is also understood to be *encoded* in path length ds constructed out of metric tensor. For the right dimension then we need to multiply ds with a factor that takes it to dimension of Planck's constant. And this dimension is energy multiplied by time period or momentum multiplied by space period. So we expect, differential Action functional along a differential path length as –

$dS = \alpha\, d\tau$.

Here we are representing Action by S and to avoid any confusion we are representing differential path length as $d\tau$ rather than ds. α is the proportionality constant that we are in search of.

Whole of spacetime period traversed by an entity as seen by an observer gets represented by *proper time* felt by the entity in its own rest frame. To come to a value that has dimensions of Planck's constant, we need to multiply this path length in time by a proportionality factor of energy units. It is natural to postulate this energy factor as *rest energy* of the entity in this *rest* frame. As this factor is to be invariant among all frames to keep Action invariant this means that proportionality factor between Action and path length will be rest energy of the entity under observation for all observers.

Thus we have proportionality constant $\alpha = m_0 c^2$ where m_0 is the rest mass of the entity and c is the speed of light. We realize that this captures physical parameter *mass* of the entity. This is welcome as we expect Action to capture whole of physics: physical parameters of entity undergoing events along with number of events. In pure gravitational phenomenon this parameter is the only physical parameter of interest. In other fields, like electromagnetic field, other parameters will also have to be included. We remark on other fields later. Currently lets concentrate only on pure gravitational fields.

If we measure length of the path in terms of space i.e. say meters, then proportionality constant should be $m_0 c$. In fact, this proportionality may be taken as the definition of rest mass. That is, rest mass is defined as proportionality constant -in conjunction with speed of light - to be multiplied with path length in continuum physics in case of pure gravitational phenomenon to give right Action.

Just a small note before we move on. We are looking at the path taken by a *ponderable* mass m_0 in presence of nothing but gravitational field. Along its worldline, given by the geodesic, events that are encountered by this entity are events due to interaction with surrounding Einsteinian vacuum (with entities of negligible mass and energy); in these events this mass overwhelms masses of any other entity involved[17]. And thus this mass captures completely the physical attributes of various events along the path. Action function along the path in an Einsteinian vacuum then captures fully physical aspects of events as well as number of events.[18] It is all that physics needs to consider.

[17] Though the test mass of the entity itself is considered small, meaning thereby not affecting the gravitational field too much, it is still a *ponderable* mass while masses of the entities in Einsteinian vacuum are 'vacuum kind' and thus really negligible.

[18] Here we are considering the case of no other fields being present – if we have electromagnetic field, for example, then multiplication factor converting number of events along the path will include charge also. That is, all physical aspects of the entity – mass, charge, isospin, spin etc. – are to be encoded in Action. These different aspects correspond to different fields, meaning thereby entities take part in events on account of these aspects; these events correspond to increments in Action that involve physical

So, with S as the Action function along a path, when an entity moves a distance $d\tau$ being measured in 'time' units, corresponding increase in S is given as: [3]

$$dS = m_0 c^2 d\tau. \qquad (4)$$

This equation is actually a postulate for us for now. We have defined Action in a certain way – capturing directly number of events along a path in multiples of Planck's constant – and this equation is connecting this definition to geometric path length measured in a particular coordinate frame through physical parameter of mass of the entity. We must interpret this relation further in terms of these two quantities – geometric path length and physical *rest energy* and see whether it coincides with orthodox treatment of these concepts. We also note that this relation *must be true in GTR, and thus in STR as well as NT.* We must look how this relation plays out in these.

Before we proceed two comments are in order here. First is that though Action is invariant fundamentally, it is sometimes written as integration of a function called Lagrangian as integrand and differential of time (as measured by the observer) as the integrating parameter (and not *proper time*). This time is off course not invariant but depends upon the observer and its chosen coordinate system. So is the Lagrangian evaluated by observer –it's not an invariant. That is, we have Lagrangian for each observer in its chosen coordinate system in such a way that its integration with parameter of time as measured by that observer gives invariant Action. That is, with S as the Action function along a path, $S = \int L dt$ or $dS = L dt$. This should be taken as the definition of Lagrangian function along a path for a given observer.

Second comment is about what Equation (4) tells us fundamentally about discrete scenario. If we could measure individual events i.e. if we could get down to discrete scenario, and if we measure, say, n events we get - $n\hbar$: $= m_0 c^2 \Delta\tau$ where $\Delta\tau$ now represents discrete time flow experienced by the entity. This means an entity can only experience time in discrete steps of - $\hbar / m_0 c^2$ – and never a flow smaller than this quantity. In continuous physics approximation when n goes to infinity and h goes to zero, this flow of proper time experienced by the entity becomes $d\tau = \frac{dS}{m_0 c^2}$.

Now let's see how this conceptualization of Action as given by equation (4) works out in NT, STR and finally in GTR. In Newtonian approximation, we take mass m (actual mass as measured by an observer) equal to m_0 and as *invariant*. In addition time is also an invariant. Also we know that distribution of events is considered as even in NT and thus we cannot have any aspect of distribution entailing a physical field. All the fields have to be *added by hand*: that is, sprinkle of events is represented by *fields over uniform grid*. Generally, a field is added by a potential function, say U. In NT it is considered as a scalar quantity depending upon the location i.e., we have $U(x)$ as potential. Scalar quantities are invariant and so is time in NT. Thus, the function Udt is invariant – so that we have the nature of dS as invariant.

Thus we get:

$$dS = m c^2 d\tau + U dt = m c^2 \left(\sqrt{(1-\beta^2)}\, dt \right) + U dt \overset{\substack{¿.inNT \\ for\beta \ll 1}}{\Rightarrow}$$

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parameters representing these aspects.

$$m c^2\left(1-\frac{1}{2}\beta^2\right)dt+Udt=mc^2 dt-\frac{1}{2}mv^2 dt+Udt.$$

β equals v/c where v is the speed of entity as observed by the observer. We approximate mass m as measured by the observer with the rest mass in NT and while applying principle of minimum action to integral of this expression of change in action we drop mc^2dt, as being a constant over all the probable paths in NT it does not enter into consideration of minimization[19]. This implies following for the value of Lagrangian:

$$S:=\int -\frac{1}{2}m_0 v^2 dt+Udt \stackrel{\text{def}}{=} \int Ldt \stackrel{\text{implying}}{\Rightarrow} L=-T+U. \qquad (5)$$

Here T is kinetic energy. Thus, our postulate leads to the standard definition of Lagrangian in NT. Equation (5) also tells us why kinetic energy in classical physics is given by the expression: $\frac{1}{2}m_0 v^2$! It tells us why *square of speed* and why factor of *1/2*.

Expression for dS above may also be written as - to introduce momentum in the expression and to get to a canonical expression –

$$dS:=-mv^2 dt+\frac{1}{2}mv^2 dt+Udt=-mvdx+Hdt.$$

Here we have dropped – as explained above - mc^2dt. And taken H as total energy (sans rest mass energy). This leads to canonical form –

$$dS:=-pdx+Hdt.$$

Here p is the momentum.

One must also remember that among various paths being considered for application of Principle of Minimum Action for a particle or for a system of particles, only the path given by this principle is physically taken. All other paths are *virtual* and only *mathematically* considered in the problem of deciding on the path of least Action.[20] Now consider not varying a path to consider virtual paths keeping the ends fixed but proceeding along the actual evolution path. Then what is being done is extending the actual physical path that is taken up by the entity and change in action comes from the variation of the length of the path at the final end point and not variation of the path itself. This change in action is counting of *new* events met along the increased evolution path.

Thus, we have S along the actual path being taken by the entity and for extension of the path length we have:

$$\delta S=p.\delta x-H\delta t. \qquad (6)$$

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[19] Though in standard treatments of the subject, this quantity m_0c^2 is neglected for good as it does not enter into minimization principle, and we have done the same here, this fact that such a quantity is a part of the Action will be important for us while drawing up parallel between action waves and spacetime waves. This quantity is multiplied by dt here and it also gives a hint that this quantity then must have interpretation as rest energy giving us most important formula in physics: $E=m_0c^2$.

[20] Once conceptual lingering problem of physics is – how does a particle sniffs various virtual paths and finally settles for the path that minimizes the action. We may do mathematics and decide path of least action, but how does a particle perform this minimization process?

p is the momentum of entity and H is the energy. (Bold face quantities are vector quantities; thus x represents three-dimensional location vector).

This differs from the above canonical form only in a negative sign and is just a matter of convention. We will stick to (6) as it is this that is generally followed in orthodox treatment of NT through Action formalism.

This expression can be interpreted as telling us how increased action along a path breaks into space and time periods realized by an observer for the entity's movement in NT. But there is subtle point to be noted here - expression in (6) does not represent total increase in Action along a path. Remember we neglected term mc^2dt from the Action calculation as it did not have impact on minimization principle. This is fine when we are comparing difference in Action along various paths. But when we are considering actual path begin taken and wish to calculate increment in total Action when entity traverses space period and time period as appreciated by an observer, we must add this term to the Action and we realize that total increment in the Action is actually -

$$\delta S = p.\delta x - (H + mc^2)\delta t = \delta S = p.\delta x - E\delta t. \qquad (6')$$

Here now E represents *total* energy including rest mass energy. This is the total increase in Action along the actual path taken by the entity as seen by the observer.

Thus, in NT, with all its approximations, we realize equation (6') means fundamentally this:

Action along an evolution path is number of events multiplied by Planck's constant. When an entity moves further along an evolution path increase in Action due to increase in events encountered breaks into space and time periods measured by an observer in proportion of functions that are defined to be momentum and total energy (including rest mass energy) values of the entity undergoing evolution by the observer.

This interpretation also feeds into our relationship between path length and Action function (4) as viewed in reference frame attached to the entity. When we are measuring path length in proper time i.e. as felt by the entity itself, there are no changes in space periods and thus Action increments get converted into only proper time increments in proportion to the energy measured in entity's own reference frame i.e. in proportion to rest energy of the entity.

We now move to STR and see how this realization plays out. In STR, along a path, for an entity we will still have $dS = m_0 c^2 d\tau$ as given above and as this Action is invariant it is so evaluated by all observers. But we have speeds reaching those of light and hence above approximations of NT are no longer valid. In STR we may proceed as (without approximating β to be very small compared to unity):

$$dS = m_0 c^2 d\tau = \left(m\sqrt{(1-\beta^2)}\right)c^2\left(\sqrt{(1-\beta^2)}\,dt\right) = m(1-\beta^2)c^2 dt = mc^2 dt - mv^2 dt : \; = mc^2 dt - mvdx = E_{total}\,dt - pdx. \qquad (6'')$$

In this derivation, we should have added, in line with NT, potential function U through the quantity U(**x**)dt; but then this gets counted in E_{total} finally. Again, we take this integration in spacetime between two fixed locations over various mathematically allowed paths and minimize it to get actual path undertaken physically.

Next, we take the physical path actually taken by the entity and extend the end points of it to calculate using this expression increment in action along this extension. That is, in STR, when we move along a physically taken path by an entity, increment in Action dS for a movement in spacetime captured by coordinate differentials dx^{μ} is given by above expression (6''). We realize that here we already have rest mass included and thus this equation is parallel to (6') for NT: just written in the language of spacetime, combining time and space components of various entities into four vectors. In STR, with all its approximations and with expression becoming more symmetric in space and time and both energy and momentum combining to make a four-vector in Minkowski spacetime, we have the same appreciation that we had in NT:

Increase in Action along an evolution path breaks into space and time periods measured by an observer in proportion to functions that are taken as momentum and total energy values of the entity undergoing evolution by the observer.

This understanding must be taken as the definitions, in NT and STR, of energy and momentum functions. What we have shown is that these definitions agree with standard definitions of these functions. In fact, we might have begun with this postulate as definition of four-vector p_{μ} and written increment in Action as:

$$dS=m_0 c^2 d\tau=p_{\mu} dx^{\mu}$$

And then move backwards to reach up to

$$dS=p_{\mu} dx^{\mu}:\ =E_{total} dt-p dx=m c^2 dt-m v dx=m c^2 dt-m v^2 dt=m\left(1-\beta^2\right) c^2 dt=\left(m\sqrt{\left(1-\beta^2\right)} c^2\right)\left(\sqrt{\left(1-\beta^2\right)}\right) dt.$$

Now due to invariance of Action we have:

$$dS=\left(p \text{¿¿} \mu\, dx^{\mu}\right)_{rest\,frame}=p_0 dx^0=E_{rest} dt_{rest}:\ =m_0 c^2 d\tau \text{¿}.$$

Combining the two expressions, if we in addition *assume*[21] symmetry in conception of mass and time we get $m_0=m\sqrt{\left(1-\beta^2\right)}$ and $d\tau=\sqrt{\left(1-\beta^2\right)} dt$. We also decipher relation equating total energy to $m c^2$.

Now we move on to case of GTR and see whether this understanding continues to hold good. Expression for action differential is exactly the same; expression for path length however now includes non-uniform metric $g_{\alpha\beta}$ rather than uniform Minkowskian metric. In GTR, thus we have Action S as

$$S=m_0 c^2 \int_A^B d\tau=m_0 c^2 \int_A^B \sqrt{g_{\alpha\beta} dx^{\alpha} dx^{\beta}}=m_0 c^2 \int_A^B \sqrt{g_{\alpha\beta} \frac{dx^{\alpha}}{dl} \frac{dx^{\beta}}{dl}} dl \qquad (7).$$

Minimization principle may now be applied to various paths between fixed locations A and B. Here l is an arbitrary parameter to describe the path between locations A and B,

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[21] This assumption is very fundamental and fact that one needs to *assume* this while working beginning with definitions of energy-momentum four-vector as proportionate function into which Action breaks into space-time four-vector is telling us something truly fundamental between these two concepts: mass and time.

and we vary the path by varying $x^\alpha(l)$ to $x^\alpha(l)+\delta x^\alpha(l)$, keeping fixed the end points, that is, setting $\delta x^\alpha=0$ at these end points. This leads to equation:

$$\frac{d^2 x^\mu}{d\tau^2}+\Gamma^\mu_{\varrho\sigma}\frac{d x^\varrho}{d\tau}\frac{d x^\sigma}{d\tau}=0 \qquad (8).$$

Here we have taken τ as any affine parameter along the path. This is the geodesic equation, which should come as no surprise. In fact, as soon as we take metric as encoding the number of events encountered and Action as proportional to number of these events along the evolution path, multiplied by physical aspects of the entity that remain constant along the path, we are assured of geodesic as the path of minimum Action for a free falling mass. This analysis however tells us that ab-initio postulate of Einstein that freely falling bodies in gravitational field shall trace a geodesic path is exactly equivalent to PMA.

Here we have not varied metric functions as we are trying to find the *path* that minimizes Action for a *given* spacetime i.e. for a given distribution of events given by metric functions. We have off course kept the coordinate system same too.

Now consider that we move *along* the path taken by the entity in GTR i.e. along the geodesic. As in NT and ST, we will get, increment in action encoding increment in number of events encountered as spacetime period traversed multiplied by $m_0 c^2$. Because of freedom to choose coordinate system arbitrarily at any step in GTR we can effect this movement along chosen path in two ways. We may either, to calculate new value of action functional in the original coordinate system, integrate along the extended portion of the path (integration limits change to coordinate values of new end point) or we may change simultaneously coordinate system in such a way that end points of integration path retain their coordinate values. In later case, integration limits remain the same in terms of coordinate values but integrand will vary due to new metric tensor values in new coordinate system being deployed. That is, new metric tensor will be different from earlier one, encoding the increase in path length or number of events along the increase:

$$g_{\alpha\beta}\to g_{\alpha\beta}+\delta g_{\alpha\beta}. \qquad (9)$$

Final physical location gets moved along the actual path taken but coordinate values are forced to be same for final locations – this is the methodology we follow for taking *Lie derivatives* on differential manifolds. The change in Action clearly will not be zero as new events are added, through variation in values of metric tensor, leading to increase in action along the path. The change in action S then is:

$$\delta S=m_0 c^2\,\delta\int_A^B d\tau=\frac{1}{2}m_0c^2\int_A^B\left(g_{\alpha\beta}\frac{dx^\alpha}{dl}\frac{dx^\beta}{dl}\right)^{-1/2}\frac{dx^\mu}{dl}\frac{dx^\nu}{dl}\delta g_{\mu\nu}\,dl=\frac{1}{2}m_0c^2\int_A^B\left(\frac{d\tau}{dl}\right)^{-1}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\left(\frac{d\tau}{dl}\right)^2\delta g_{\mu\nu}\,dl=\frac{1}{2}m_0c^2\int_A^B$$

. (10)

Here we have used l as a dummy parameter in between steps and we need to take c^2 inside to convert coordinate differentials dx^μinto dimensions of space and not time. Comparing this equation with definition of energy-momentum tensor (6'') gives us this tensor having standard classical definition of energy and momentum, combined into one tensor in relativistic scenario of Einstein's. In fact, this is the reason why we have standard definitions of energy and momentum as what these are! These are the expressions for proportionality functions by which increment in Action breaks into space and time periods for an observer. Equation (10) tells us basis of standard definitions of energy and momentum.

Change in Action S, due to change along the path taken by the entity then is captured by equation (3) exactly parallel to (5) and (6)! Let's summarize what we have deciphered so far:

1. *Action is number of events encountered along an evolutionary path multiplied by Planck's constant. All observers observing the same entity observe same number of events that it encounters along its evolution path between any two locations on it and thus have invariant Action functional. In continuous physics approximation of classical domain we have Planck's constant moving towards zero while number of events move towards infinity, to give the multiple as a real function of dimensions of Action.*

2. *To realize what path will be chosen among infinite ones possible between given end points, we deploy principle of minimum action and choose the path as physical evolution path for which variation of action compared to near ones is zero.*

3. *For estimating changes in the Action, one needs to move along actual worldline taken by the entity and encounter new events. Various observers, depending upon their relative motions and chosen coordinate systems for measurements, divide observed increment in this action functional into appreciation of different space and time periods traversed by the entity in proportion to functions that they record as momentum and energy tensor.*

Now we move to application of our understanding developed so far to represent Action in GTR as a *field theory*.

To get to the EFE that would tell us how geometric quantities shall be connected to mass energy tensor in a field or spacetime region, instead of a path of a single entity now we consider a collection of entities in an extended region of spacetime. We will follow exactly the same prescription that we adopted for analyzing action along an individual evolution path (we remind ourselves that we are presently considering only pure gravitational field, making only rest masses of entities as meaningful physical parameters): Action would be equated to number of events in a region multiplied by Planck's constant and then we will vary Action as this volume expands and relate this increment through equation (3) to energy-momentum tensor. Again, in continuum physics of EFE Action will assume real values indicating number of events in the region encoded through expressions involving metric tensor components.

Interaction between different entities in a given region works through the modification of the metric tensor by creating events that this tensor encodes. Matter is modifying the geometry. And entities follow the geodesics determined by metric tensor. Geometry is directing the matter. That's the way interaction affects the motion of the entities.

Observer observes entities to be moving along their individual timelike geodesics in the given spacetime. Observer draws up a coordinate system, foliating whole of spacetime using spacelike hypersurfaces as leaves, each having a given time coordinate value 't'.

On individual hypersurfaces observer chooses a three-space coordinate system, with only requirement that this system varies smoothly while moving from one hypersurface to another.[22]

Consider the sandwich between two hypersurfaces– indicated by time coordinate values (arbitrary, completely arbitrary parameters) t_i and $t_f = t_i + \partial t$. Observer chooses an arbitrary region on initial hypersurface and calculates number of events that are encountered by all the entities emanating from this region while moving along these geodesics between two hypersurfaces. This number gives action in the spacetime region up to a multiple of Planck's constant in a discrete scenario. Question that we need to answer now is - in continuous scenario of classical physics, what geometric object represents this number?

We assume that geodesics that emanate from space hypersurface from various locations do not cross each other in region under consideration i.e. do not cross before they reach the next hypersurface. Finally we are going to take the limit going to an infinitesimal volume about a point location and thus this assumption is fine. These geodesics do however converge or diverge depending upon the spacetime curvature. This convergence or divergence of geodesics is the crux of gravitation: gravitation works or shows its existence through these phenomena.

This convergence or divergence however affects the volume that is encompassed within these geodesics. However, one may appreciate that *number* of geodesics emanating from a given three-dimensional region of the first hypersurface will remain same when we reach the next hypersurface. This number of geodesics is the representative of number of entities under observation. Off course, in continuum approximation we have a real number representing this 3D volume on initial spacelike hypersurface, but we realize that in reality, however humungous, it's an integer number. When these geodesics reach next spacelike hypersurface 3D volume that these landings correspond to, again, is a real number in continuum approximation and though this volume may be different in real number approximation – this volume is given by coordinate volume multiplied by square root of modulus of determinant of matrix composed of three-dimensional metric components. We however appreciate that, though these two real numbers, representing initial volume on initial spacelike hypersurface and final volume on final spacelike hypersurface are different, *number* of geodesics emanating from initial hypersurface and reaching final hypersurface must be same. We should keep this in mind while calculating the number of events that are encompassed in this sandwich volume.

Again, along any geodesic i.e. for any entity under observation, total number of events that occur is the same between two spacelike hypersurfaces. This point is due to the fact that all the entities are participating in the evolution of spacetime and are under going equal number of events as all events are being communicated to all. This follows or this situation is enforced due to subtle impact of fact that all observers observe all events. This later fact itself follows or is enforced due to fact that we preserve all events in any region while going from one coordinate system to another. This subtle point is going to

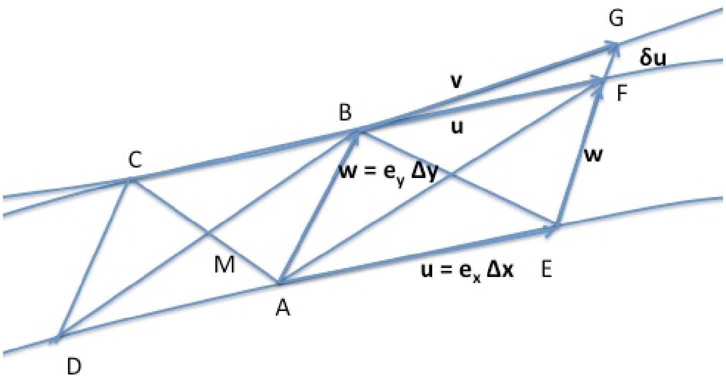
[22] It is clear that parameter 't' can be chosen any which way with only condition is of monotonic increase. Similarly, space coordinates can be chosen quite arbitrarily. It is well known than in classical mechanics, one can re-parameterize the absolute time t in such a way that problem become invariant to arbitrary choosing of time parameter. It is done to bring time t at par with other coordinates 'q'. It leads to a compromise of increasing the parameters by one and to a situation where Hamiltonian is zero. ADM [] calls GTR already parameterized because GTR has arbitrariness in selection of time and space coordinates built in.

be elaborated in next section in great detail and is one of the most fundamental aspect of spacetime of GTR that goes without much appreciation.

Thus, basic geometric volume of the region – given by $\sqrt{g}\,dx^0 dx^1 dx^2 dx^3$ - between two hypersurfaces is not the real measure of the number of events but we will have to modify this by the volume of distortion due to convergence or divergence of the geodesics. Here enters the Riemannian Scalar Curvature.

Let's then see how divergence or convergence of geodesics affects the volume. To get to a representative geometric quantity for this, we will be taking a four-dimensional volume approximating like a cube with coordinates of typical corners as $\left(t_0 \pm \frac{1}{2}\Delta t, x_0 \pm \frac{1}{2}\Delta x, y_0 \pm \frac{1}{2}\Delta y, z_0 \pm \frac{1}{2}\Delta z\right)$. [MTW – Box 15.1]. There are 16 corners for such four-dimensional cubical region, with boundary consisting of eight three-dimensional cubical portions of hypersurfaces. Each of these three-dimensional cubical regions have six two dimensional faces, but these faces never *face* outward as these are glued to each other. We take all the edges of this four-dimensional cubical region as geodesics, and tangents to them as coordinate axes or *parallel* thereof.

We first take a two-dimensional face spanned by two nearby geodesics to understand how divergence or convergence of these affects area that is enclosed. That is, we are looking at how a parallelogram drawn in a curved region using geodesics measures area in comparison to that would be encompassed by curves parallel to two geodesics emanating from a corner. We remind ourselves that we are going to take limit tending to zero of the volume and that our construction becomes more and more exact.



In the picture above we consider first a geodesic DAE. We take x as an affine parameter along this geodesic. Tangent vector to this geodesic is taken as coordinate vector $\mathbf{e}_x$.

We choose a location A on geodesic DAE and assign it coordinate value x = 0. Another location B is selected in the vicinity and draw up a geodesic between A and B. B is assigned same coordinate value of x =0 and tangent to geodesic is taken as coordinate vector $\mathbf{e}_y$. We then use the Schild's ladder construction [MTW – Box 11.6] to draw a worldline CBF that can be considered parallel to DAE. For this we take a small change in x as Δx (towards negative $\mathbf{e}_x$) and mark location D on our geodesic. We then draw a geodesic between D and B, take a location M half way through and then draw a geodesic from A towards M and then extending it beyond by same distance as between A and M to reach a location C. Similarly we do it for a some distance Δx towards positive $\mathbf{e}_x$ to reach E and then follow the above stated construction to reach location F. Curve CBF can then be considered as parallel to geodesic DAE. This off course is not a geodesic through B.

Now consider an actual geodesic between C and B and extending further – we mark location at a distance equal to parameter Δx on this as G. FG is the vector that gives the deviation of actual geodesic from the parallel curve that we have drawn to DAE.

When we use integral of area we get the area as that of ABGE; we should have the area ABFE=area ABCD as the one representing the number of geodesics multiplied by number of events on them – that is devoid of any divergence or convergence deviations which gets represented by area BFG.

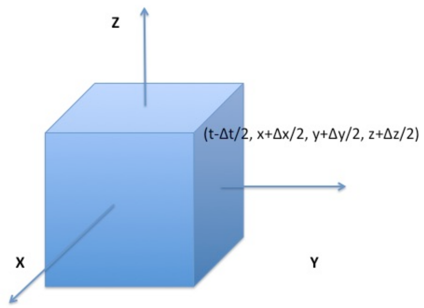
If we take vector **u = e$_x$Δx** pointing from B to F along the close loop BCDAB we will reach the vector **v** pointing from B to G and the rotation δu^{α} (α varying from 0 to 3) depends directly upon Riemannian curvature components R^{α}_{xyx}. We also geometrically immediately see that the deviation area BFG depends *linearly* on δu^{α} and thus so on curvature component R^{α}_{xyx}. This direct relation between rotations caused along a loop and the deviation in the area of the geodesic parallelogram is the crux of this analysis.

In this analysis we have used geodesic EF exactly parallel to AB (Schild's construction). But there is going to be a divergence or convergence between these two and this will also cause a rotation of a vector and also deviation in the area and those should depend upon R^{α}_{yxy}.[23] Thus we have rotations of vectors along the boundary as well as area of the geodesic parallelogram directly depending upon the corresponding Riemannian curvature components; and the two enjoying a direct correlation.

Now we move to three-dimensional (hypersurface) cube to realize relation between deviation in the three-dimensional volume due to divergence or convergence of geodesics. We take this cube as a boundary cube of four-dimensional cube with corners $\left(t_0 \pm \frac{1}{2}\Delta t, x_0 \pm \frac{1}{2}\Delta x, y_0 \pm \frac{1}{2}\Delta y, z_0 \pm \frac{1}{2}\Delta z\right)$ and use Riemannian normal coordinates. [2]

The rotation associated with front face is: $e_{\lambda} \wedge e_{\mu} R^{|\lambda\mu|}_{yz} \Delta y \Delta z$. Here indices within vertical strokes get summed up with restriction of λ<μ. We similarly take rotation associated with the back face and take the *moment of rotation* (E. Cartan [2]): $\left(P_{centeroffrontface} - P\right) e_{\lambda} \wedge e_{\mu} R^{|\lambda\mu|}_{yz} \Delta y \Delta z.$ Here $P_{\text{center of front face}}$ is, as obvious central location on front face and P is any location in the spacetime (when we add up all moments of rotations to get resultant one, location of P becomes immaterial as total of all rotations is zero.)

[23] Also as is well known for loop to completely close we will also need a component of commutation between two vectors **e$_x$** and **e$_y$**. As we have taken these to be coordinate axes this will be zero but in general case of arbitrary vectors as tangents to geodesics forming geodesic parallelogram we will have this commutation between two vectors coming in. This is the reason why Riemannian curvature tensor includes covariant derivative along commutation of two vectors.



When we add moment of rotation of front and back faces (about the same but arbitrary location P) we get *net moment* of the rotation for these two faces: $e_x \wedge e_\lambda \wedge e_\mu R^{|\lambda\mu|}_{yz} \Delta y \Delta z \Delta x$. With the contribution from all the six faces we get final net moment of rotation:

$$e_x \wedge e_\lambda \wedge e_\mu R^{|\lambda\mu|}_{yz} \Delta y \Delta z \Delta x + e_y \wedge e_\lambda \wedge e_\mu R^{|\lambda\mu|}_{zx} \Delta y \Delta z \Delta x + e_z \wedge e_\lambda \wedge e_\mu R^{|\lambda\mu|}_{xy} \Delta y \Delta z \Delta x. \quad (11)$$

This sum is action of 3-form - $e_v \wedge e_\lambda \wedge e_\mu R^{|\lambda\mu|}_{|\alpha\beta|} d x^v \wedge d x^\alpha \wedge d x^\beta$- on three-dimensional volume element $e_x \wedge e_y \wedge e_z \Delta y \Delta z \Delta x$. With our earlier analysis of two dimensional faces leading to rotations of kind $e_\lambda \wedge e_\mu R^{|\lambda\mu|}_{yz} \Delta y \Delta z$ representing deviation in area, we immediately realize that expression (11) is nothing but deviation in volume of three dimensional cube over straightened (in a Minkowskian sense) cube. That is we realize that *moment of rotation* (as envisaged by E. Cartan [2]) is a representative of modification to basic volume calculated on the basis of metric tensor to come to number of spacetime coincidences that are to be found in the region.

We may now move to four-dimensional cube. Moving exactly parallel to our movement from two-dimensional geodesic parallelogram to three-dimensional geodesic cube, we may appreciate that, for each small enough cube representative expression for modification over basic volume expression for coming to number of spacetime coincidences contained within the volume is given by (now a scalar): $R\sqrt{g}\,\Delta x \Delta y \Delta z \Delta t$. This becomes exactly true as we go towards infinitesimal volume element and then integrate over the whole region to get to the representative expression for number of spacetime coincidences i.e. events contained therein –

$$S = \int (R+\sigma)\, d\tau = \int (R+\sigma)\sqrt{-g}\, dx^0 dx^1 dx^2 dx^3. \qquad (12)$$

Though this expression should contain proportionality factors while equating right hand sides to Action at left hand side, physical parameters (masses in case of pure gravitational fields under consideration) in these get cancelled by energy-momentum tensor in (3) (equivalence principle) while rest get absorbed in constant κ. σ is some constant for proportionality to regular volume differential. If R is negative we have volume overestimating the events contained in the zone while if R is positive volume is underestimating the volume. We then decide, on this, whether σ is positive or negative.

This additional term of σ gives rise to term $\lambda g^{\alpha\beta}$ where λ is *cosmological constant*. This term was not there in the original EFE (1) but was introduced sometime later by Einstein as a desperate attempt to get to a *stable* universe. After realization of expanding universe, he termed this introduction as his biggest blunder. This term however has come to be realized as a necessary addition for EFE. Thus, EFE takes the form as -

$$R^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} R = \kappa T^{\alpha\beta} + \lambda g^{\alpha\beta}. \qquad (1A)$$

Thus, we realize that even in GTR as a field theory Action does come out as number of events multiplied by Planck's constant, when we take these numbers to be infinitely large and Planck's constant to go towards zero to get to expression suitable for continuous spacetime picture of classical physics.

We may proceed from (11) above, via extension of the region along the geodesics emanating from a spatial hypersurface i.e. extending beyond upper hypersurface to go to another one and then redrawing the coordinate map to keep the coordinate values same for final hypersurface locations of geodesics, leading to variation in metric tensor $g_{\alpha\beta} \rightarrow g_{\alpha\beta} + \delta g_{\alpha\beta}$ rather than change of coordinate values of boundaries of integration and the reach EFE via defining expression of (3) for energy-momentum tensor.

The whole analysis above gives us confidence about our understanding of Action as number of events in a region or along a path multiplied by the Planck's constant. We proceed with this understanding in rest of the monograph. But before we move on to next section, we must say something about fields other than gravitational field, including our familiar electromagnetic field. We have derived EFE in simplistic scenario of having no other field present in the spacetime region. What happens when other fields are present?

To appreciate this, let's take electromagnetic field as an example. A subtle point to note is that an observer, in any general treatment of the subject is always - though unknowingly in most of the cases - taken to be electrically *neutral*. In fact, it is always taken to be neutral in terms of any other *charge* responsible for any interaction other than the *universal* interaction of gravitation for which the *charge* is *mass*. Observer then has only mass as physically relevant parameter and is subjected only to gravitational interaction. This means fundamentally that all the interactions that require *charges* other than mass are not available to the observer to appreciate directly. For example, consider presence of an electric charge in any location on the spacetime. It interacts with other charges in the spacetime through electromagnetic interactions but not with the observer that is electrically neutral.

If an observer cannot appreciate events that some other entity is undergoing, it is evident its calculation of Action will go haywire for this entity and it is going to realize that the entity is not following the geodesic calculated in terms of minimizing the Action on the basis of its estimated metric tensor. This will force the observer to conceptualize presence of a field that it itself is oblivious to as well as a charge that entity deviating from the geodesic must possess corresponding to its behavior under this field. *Thus, it models such fields over the spacetime in addition to metric tensor that it has conceptualized for the universal gravitational field.*

What about possibility of an electrically *charged* observer able to encode electromagnetic field in the spacetime structure like neutral observer doing it for gravitational field? To see how different electromagnetic - and other non-gravitational - fields are from gravitation, let's first recapitulate why has gravitational influence of a body of mass over other bodies of masses been able to be encoded in spacetime fabric. The reason off course is Einstein's equivalence principle: mass of a body that goes to determine force it feels in a gravitational field is equivalent to mass of the body that goes into its kinematical reaction to application of any force. In our understanding drawn till now, it means that mass that goes into decision on number of events that a body will undergo at a location in spacetime due to gravitational field - thereby deciding Action thereat - is same as mass that appears in the energy-momentum tensor in proportion to components of which Action breaks spacetime distance into space and time periods. Thus this mass cancels out on either side giving a relation, called EFE, between expression derived from metric function and energy-momentum tensor, without any reference to mass of body at that location. And hence whole of equation has no physical parameter belonging to the test body at any location. That's the meaning of gravitational field being encoded in the spacetime: its value at any location on spacetime can be calculated without any consideration of what test body may be put there.

Now consider electromagnetic field – or any field other than gravitational field. An entity's reaction to this field depends upon a physical parameter – generally called a *charge* – belonging to the entity other than the mass. In fact, these fields are created by the entities that have this charge and affect only those entities that have this charge. Thus number of events the electromagnetic or some other field creates along with the entity at that location, giving us value of Action function at that location, depends upon charge of the entity and when this Action function increment creates space and time periods in proportion to energy-momentum tensor, value of this parameter remains in the equation. Components of energy-momentum tensor depend upon only the charges of the entities creating these and the mass of the test body thereat. We have a ratio of mass to charge for the test body appearing on one side of the equation and geometric quantities on other. *We cannot say that field is encoded in the spacetime fabric as metric tensor components are not independent of charge of the entity placed at a location.*

Each entity that is charged necessarily has mass too – and when it moves under the influence of electromagnetic field (i.e. under the influence of events caused by other entities that are charged) – its mass participates in the events that are gravitational kind from the new location and effectively – unless we take it to be a passive situation where test particle is of negligible nature and does not substantially affect the spacetime curvature – changes the spacetime fabric. Thus, finally electromagnetic field also works back on gravitational field by modifying locations of charged particles.

Thus, we add energy-momentum tensor of electromagnetic field on right hand side of EFE (2) as it acts upon the gravitational field. That is, though an electric charged mass has moved under influence of electromagnetic field (in addition to gravitational field), its movement has caused changes in energy-momentum distribution. This works back on metric tensor that is conspicuous to observer. However, while calculating motion of a charged mass, at any instant, it has to use a force term on right hand side of the geodesic equation (that involves charge and mass) to account for events due to electromagnetic field inconspicuous to neutral observer.

We have, in our interpretation, by stating that if charged particles are present that can participate in photon exchange, we have electromagnetic field, relegated field to of secondary importance and charged particles are primary. However, in most of the cases, we are concentrating on a region in spacetime and there are charges present outside this region also. Sum total of the charges outside this region are represented mathematically in form of electromagnetic field. That is, effects created due to *outside* charges are represented in terms of fields. For charged particles inside the region we directly encode their effects without going through their fields - off course, we can also convert effect of all the charges into fields and then have only fields, but this is just mathematical jugglery. We can choose mathematical representation by convenience.